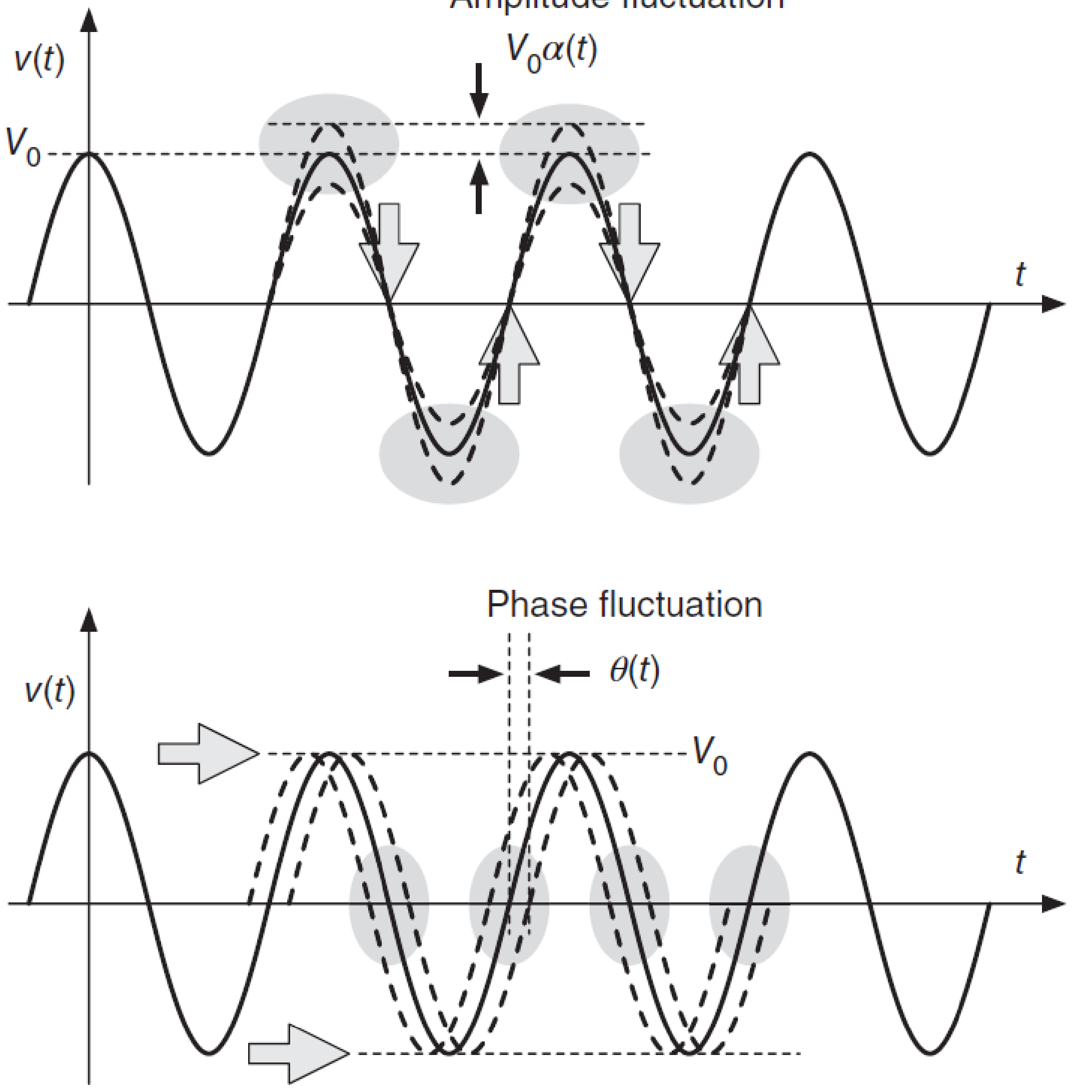
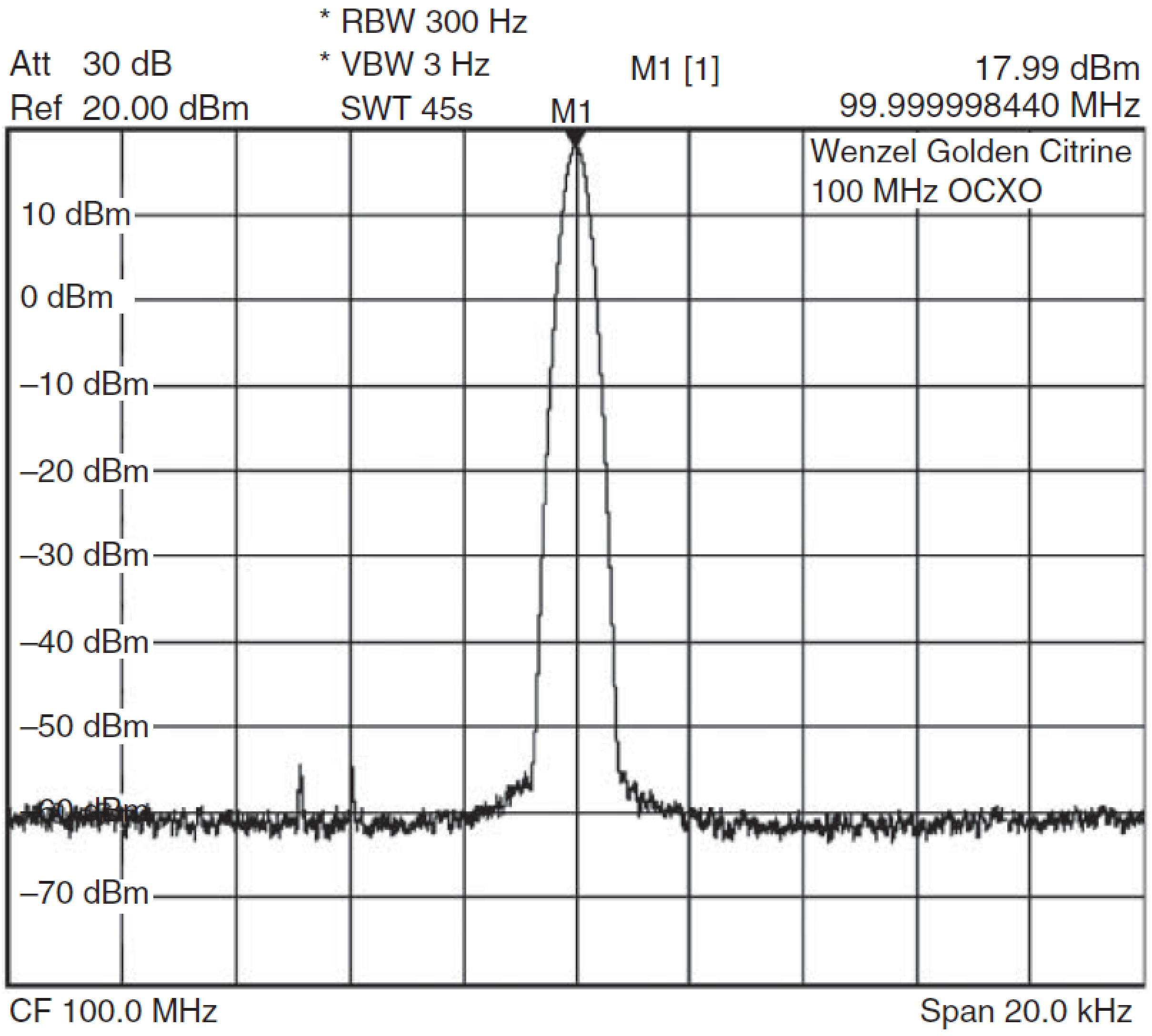
Лекция 4. Математическое описание фазовых шумов генераторов

Мишагин К.Г.

Амплитудный и фазовый шум, спектр мощности синусоидального сигнала





Ulrich L. Rohde, Enrico Rubiola, and Jerry C. Whitaker. *Microwave and Wireless Synthesizers: Theory and Design*, Second Edition. Chapter 2. ALMOST ALL ABOUT PHASE NOISE

Модель сигнала прецизионного генератора

$$V(t) = [V_0 + \varepsilon(t)] \sin[\underbrace{2\pi f_0 t + \varphi(t)}_{\Theta(t)}]$$

$$\left|\frac{\varepsilon(t)}{V_0}\right| \ll 1 \qquad \left|\frac{1}{2\pi f_0}\frac{d\varphi}{dt}\right| \ll 1$$

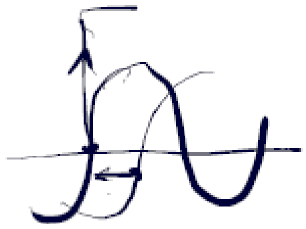
Мгн. знач. частоты

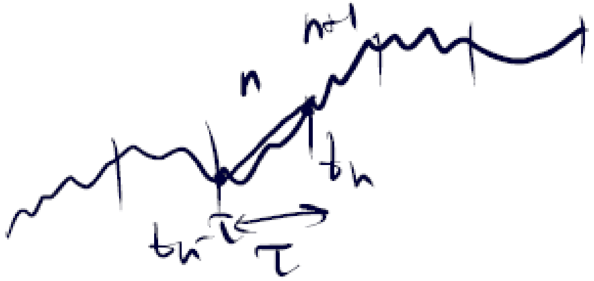
$$f(t) = \frac{1}{2\pi}\dot{\Theta} = \frac{1}{2\pi}\frac{d}{dt}[2\pi f_0 t + \varphi(t)] = f_0 + \frac{1}{2\pi}\frac{d\varphi(t)}{dt}$$

отн. откл. частоты

$$y(t) = \frac{f(t) - f_0}{f_0} = \frac{1}{2\pi f_0}\frac{d\varphi}{dt} = \frac{dx}{dt}$$

$$x(t) = \frac{\varphi}{2\pi f_0}$$ — фаз. время





$$y_n(\tau) = \frac{1}{\tau}\int_{t_n-\tau}^{t_n} y(t)\,dt = \frac{x(t_n) - x(t_{n-1})}{\tau}$$

$t_n - \tau$

Представление сигнала в частотной области

$$x(t) \xrightarrow{F} F_x(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$E_x = \int_0^T x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F_x(\omega)|^2 d\omega$$

$$P_x = \frac{E_x}{T} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|F_x(\omega)|^2}{T} d\omega$$

$T \to \infty$:

$$S_x(\omega) \triangleq \lim_{T \to \infty} \frac{|F_x(\omega)|^2}{T}$$

$S_x(\omega)$, $R[\tau]$

Т Винера-Хинчина

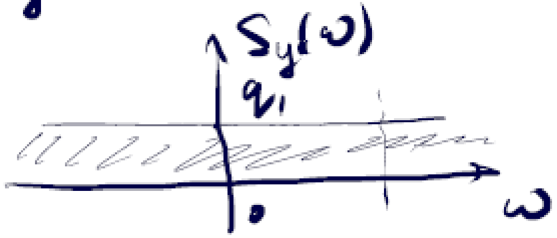
$$S_x(\omega) = \int_{-\infty}^{\infty} R[\tau] e^{-j\omega\tau} d\tau$$

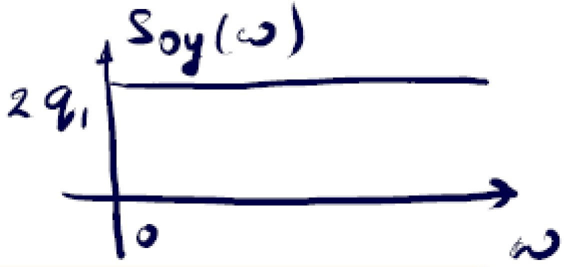
$$R[\tau] = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) e^{j\omega\tau} d\tau$$

Модель сигнала прецизионного генератора

$$\langle \xi(t)\,\xi(t+\tau)\rangle = \underbrace{q_1\delta(\tau)}_{R_\xi(\tau)} \rightarrow S_y(\omega) = q_1$$

→





$$S_x(\omega) = \frac{S_y(\omega)}{\omega^2}$$

$$\dot{x}(t) = y(t)$$

$S_x(f) = S_y(f)/(2\pi f)^2$,

where $S_x(f)$ =

PSD of the time fluctuations, sec²/Hz.

$S_\phi(f) = (2\pi\nu_0)^2 \cdot S_x(f) = (\nu_0/f)^2 \cdot S_y(f)$

PSD of the phase fluctuations, rad²/Hz and its logarithmic equivalent £(f), dBc/Hz.

$$£(f) = 10 \cdot \log[½ \cdot S_\phi(f)]$$

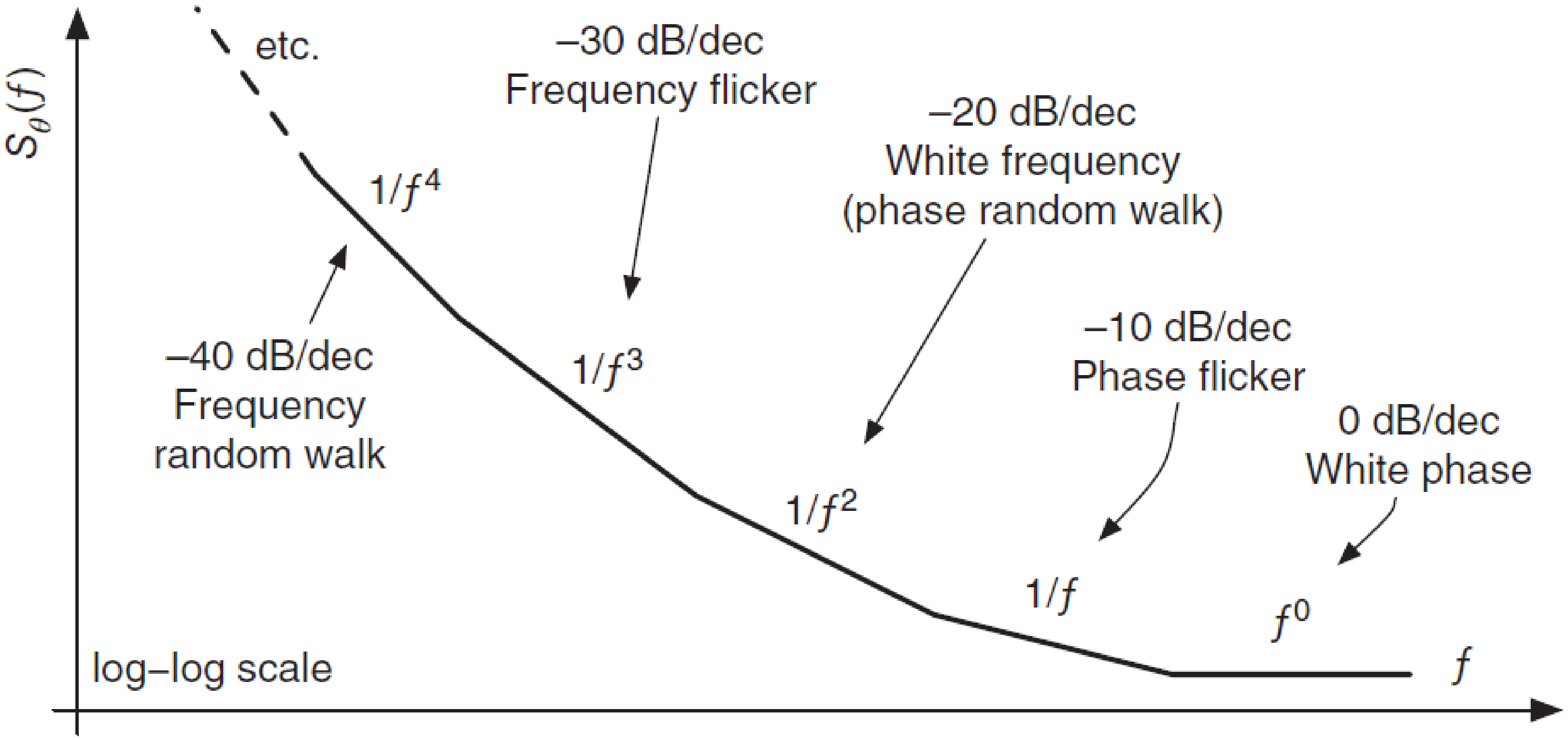
$S_y(f) = h(\alpha)f^{\alpha}$,

where:			
	$S_y(f)$	=	one-sided power spectral density of y, the fractional frequency fluctuations, 1/Hz
	f	=	Fourier or sideband frequency, Hz
	$h(\alpha)$	=	intensity coefficient
	α	=	exponent of the power law noise process.

The most commonly encountered noise spectra are

White (f^{0})
Flicker (f^{-1})
Random Walk (f^{-2})
Flicker Walk (f^{-3}) .

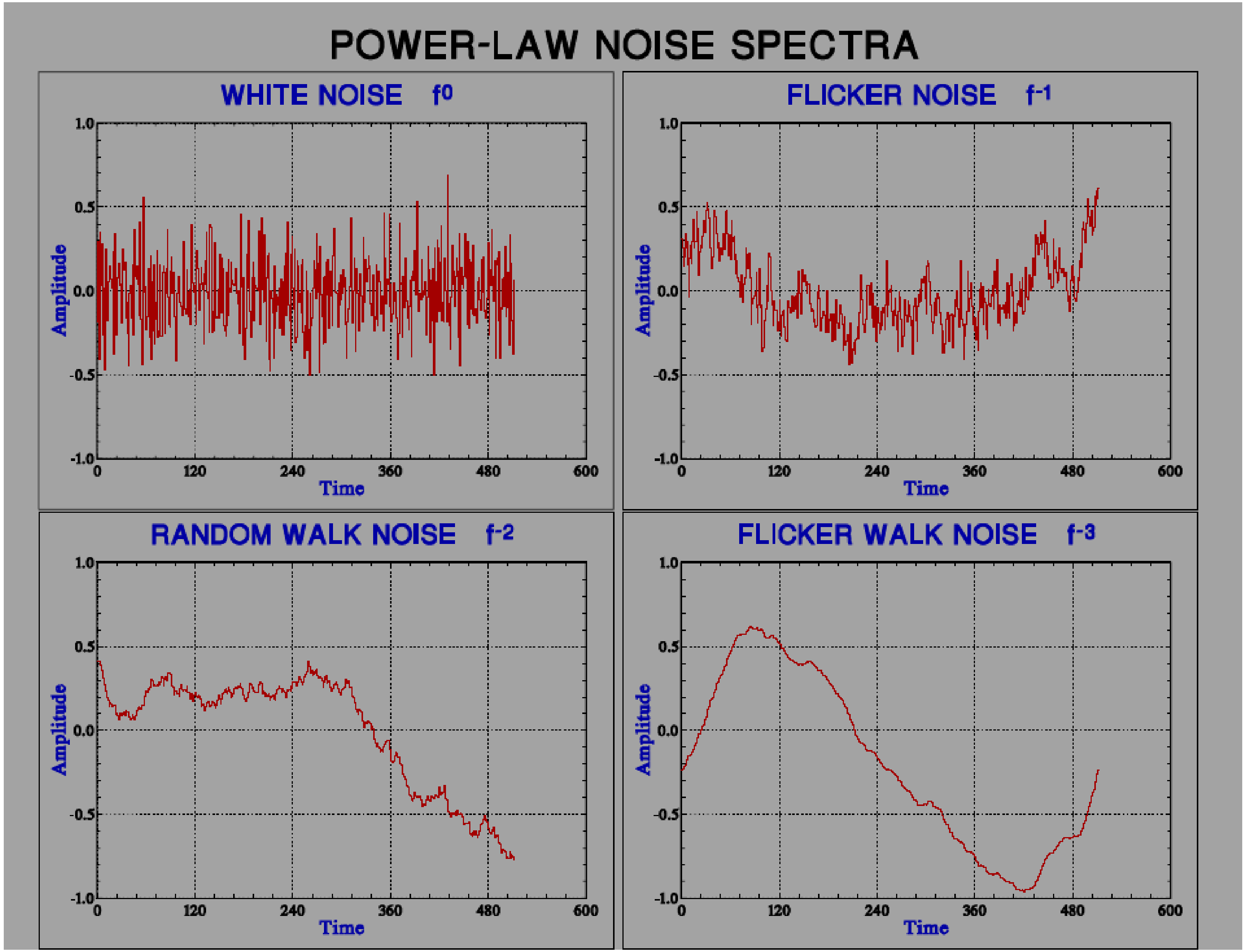
Модель сигнала прецизионного генератора



Noise type	Phase noise, $S_\theta(f)$	
	Law	Unit
White PM	b_0	rad^2/Hz
Flicker PM	$\frac{b_{-1}}{f}$	rad^2
White FM	$\frac{b_{-2}}{f^2}$	$rad^2\ Hz$
Flicker FM	$\frac{b_{-3}}{f^3}$	$rad^2\ Hz^2$
Frequency RW	$\frac{b_{-4}}{f^4}$	$rad^2\ Hz^3$

Ulrich L. Rohde, Enrico Rubiola, and Jerry C. Whitaker. *Microwave and Wireless Synthesizers: Theory and Design*, Second Edition. Chapter 2. ALMOST ALL ABOUT PHASE NOISE

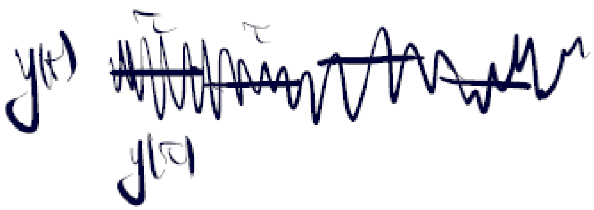
Примеры разных типов шума



Handbook of Frequency Stability Analysis

W.J. Riley
Hamilton Technical Services

Характеристика частотного/фазового шума во временной области

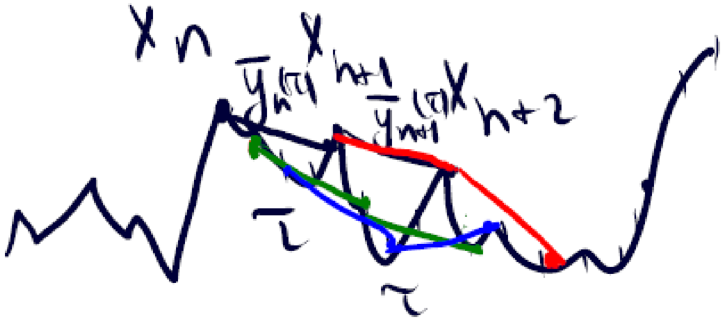


$$I(\tau) = \langle y^2(\tau) \rangle \Rightarrow \frac{1}{N-1}\sum_{j=1}^{N}\left(y_j - \frac{1}{N}\sum_{i=1}^{N} y_i\right)^2$$

$$I(\tau) \to \infty$$

$$\sigma_y^2(\tau) = \frac{1}{2}\left\langle \left(y(t+\tau) - y(t)\right)^2 \right\rangle$$

СКДО, вариация Аллана



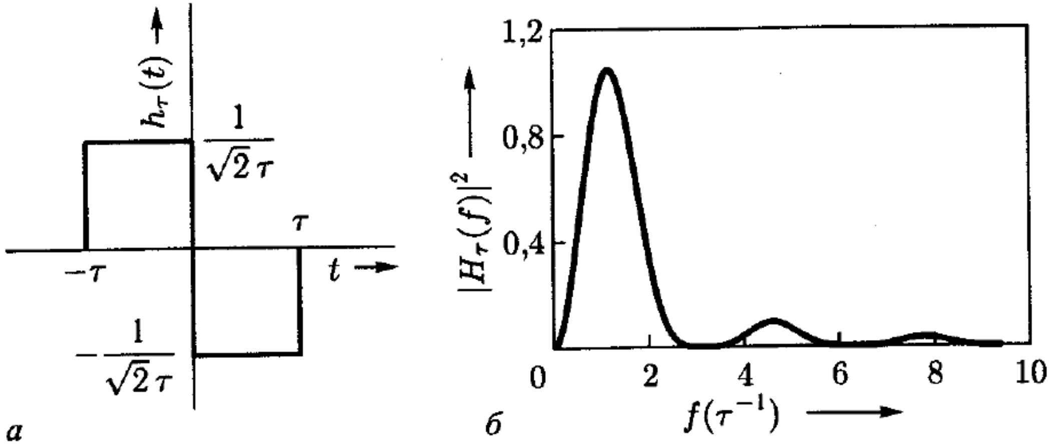
$$\hat{\sigma}_y^2(\tau) = \frac{1}{2(N-2)}\sum_{n=1}^{N-2}\sigma_n^2(\tau)$$

$$\sigma_n(\tau) = y_{n+1}(\tau) - y_n(\tau) = \frac{1}{\tau}\left[X_{n+2} - 2X_{n+1} + X_n\right]$$

Связь между характеристиками в частотной и временной области

$$\sigma_y^2(\tau) = \frac{1}{2}\langle(\bar{y}_2 - \bar{y}_1)^2\rangle = \left\langle \frac{1}{2}\left(\frac{1}{\tau}\int_t^{t+\tau} y(t')\,dt' - \frac{1}{\tau}\int_{t-\tau}^{t} y(t')\,dt'\right)^2 \right\rangle$$

$$\sigma_y^2(\tau) = \left\langle \left(\int_{-\infty}^{\infty} y(t')h_\tau(t-t')\,dt'\right)^2 \right\rangle \Rightarrow \sigma_y^2(\tau) = \int_0^{\infty} |H_\tau(f)|^2 S_y^{\text{1-sided}}(f)\,df$$



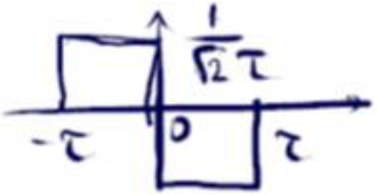
$$|H_\tau(f)|^2 = 2\frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2}$$

$$\sigma_y^2(\tau) = 2\int_0^{\infty} S_y(f)\frac{\sin^4(\pi f \tau)}{(\pi f \tau)^2}\,df.$$

Ф. Риле Стандарты частоты

Связь между характеристиками в частотной и временной области

$$\sigma_y^2(\tau) = \frac{1}{2}\langle (y_2-y_1)^2\rangle = \left\langle \left| \int_{-\infty}^{\infty} y(t')\, h_\tau(t-t')\, dt' \right|^2 \right\rangle$$



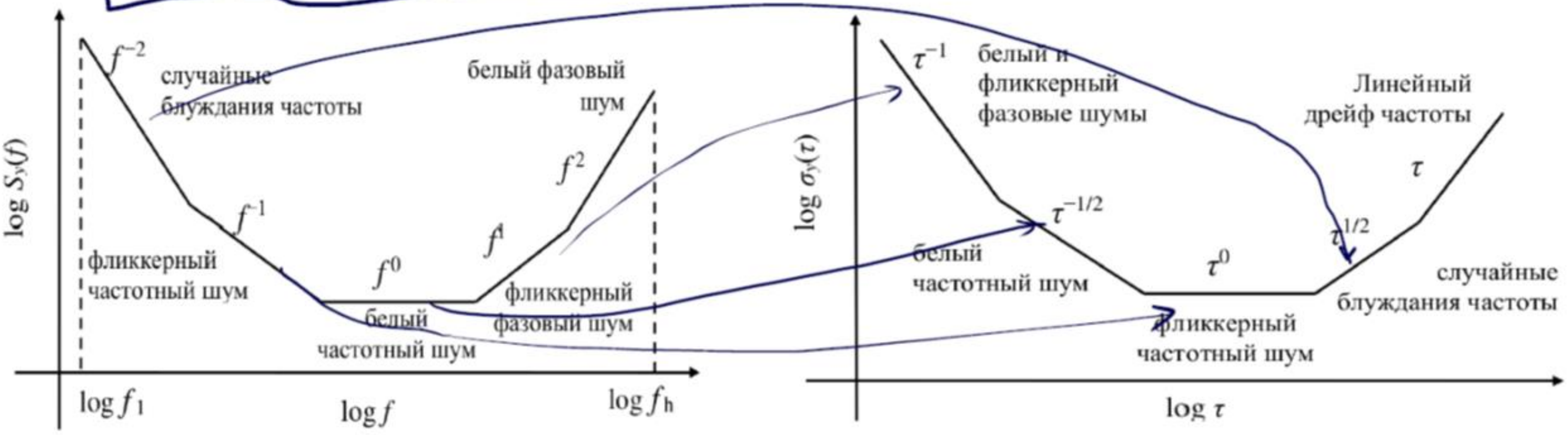
$$S_y(f) = \int R_y[\tau]\, e^{-i2\pi f\tau}\, d\tau$$

$$R_y(\tau) = \langle y(t)\, y(t+\tau)\rangle, \quad |H_\tau(f)|^2 = 2\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}$$

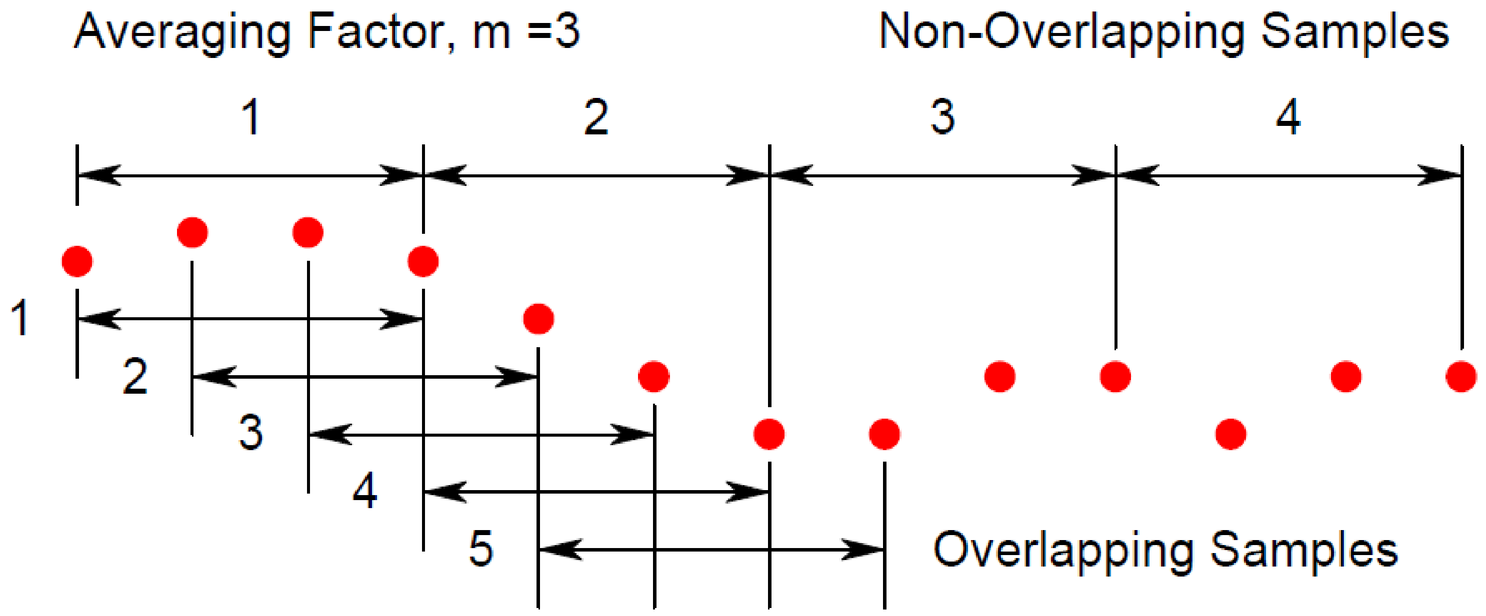
$$\sigma_y^2(\tau) = 2\int_0^{f_h} S_y(f)\, \frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}\, df$$

$$S_y(f) = \sum_{\alpha=-2}^{2} h_\alpha f^\alpha \longrightarrow \sigma_y^2(\tau) \sim \tau^\mu$$

$$\mu = -(\alpha+1), \ \alpha<2$$



Вариация Аллана с перекрытиями



Non-Overlapped Allan Variance: Stride = τ = averaging period = m·τ_0

$$\sigma_y^{2}(\tau) = \frac{1}{2(M-1)} \sum_{i=1}^{M-1} \left(y_{i+1} - y_i\right)^2$$

Overlapped Allan Variance: Stride = τ_0 = sample period

$$\sigma_y^{2}(\tau) = \frac{1}{2m^2(M-2m+1)} \sum_{j=1}^{M-2m+1} \sum_{i=j}^{j+m-1} \left(y_{i+m} - y_i\right)^2$$

Вариация Адамара и модифицированная вариация Аллана

$$H\sigma_y^2(\tau) = \frac{1}{6(M-2)} \sum_{i=1}^{M-2} \left[y_{i+2} - 2y_{i+1} + y_i \right]^2$$

$$\mathrm{mod}\,\sigma_y^2(n\tau_0) = \frac{1}{2\tau^2} \left\langle \left[\frac{1}{n} \sum_{i=0}^{n-1} x_{i+2n} - 2x_{i+n} + x_i \right]^2 \right\rangle$$

$$\mathrm{mod}\,\sigma_y^2(n\tau_0) = \frac{1}{2n^4\tau_0^2(N-3n+1)} \sum_{j=0}^{N-3n} \left\{ \sum_{i=j}^{j+n-1} x_{i+2n} - 2x_{i+n} + x_i \right\}^2$$

Девиация временного интервала, модель ухода часов (шкалы времени)

МОВИ

ДВИ, TDEV

TVAR → $\sigma^2_x(\tau) = (\tau^2/3)\cdot Mod\ \sigma^2_y(\tau)$.

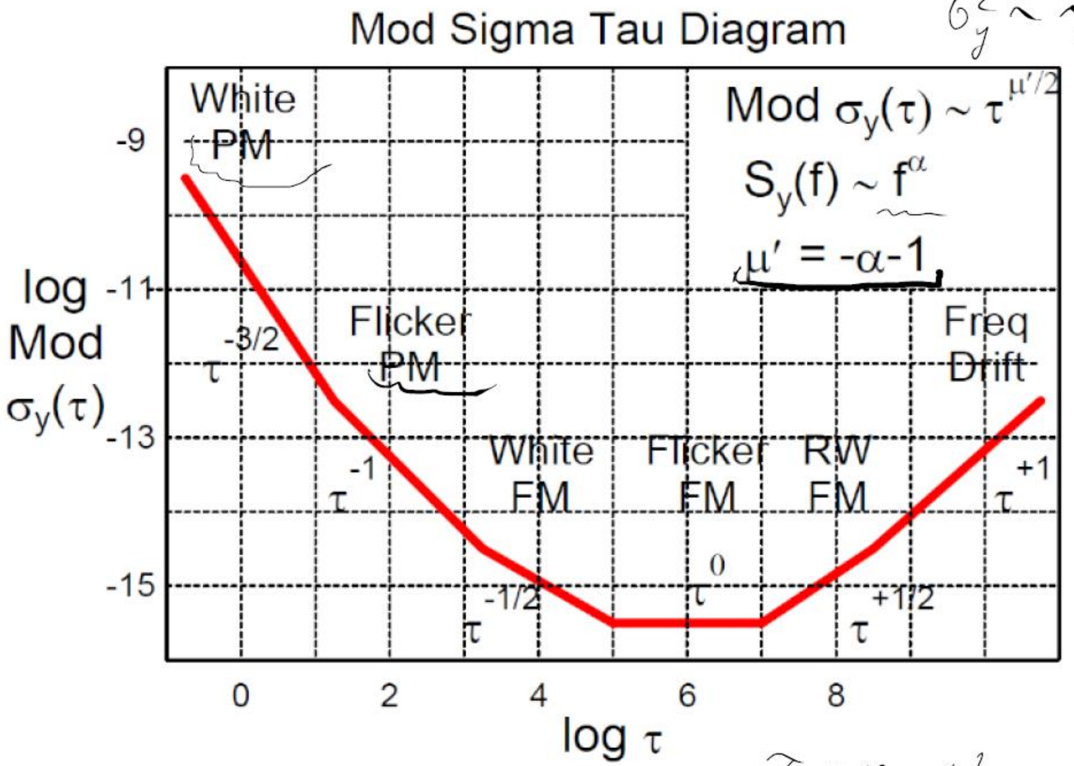
БФШ

$\sigma_y^2 \sim \frac{1}{\tau^2}$

$\Downarrow$

$\sigma_x^2 = const$

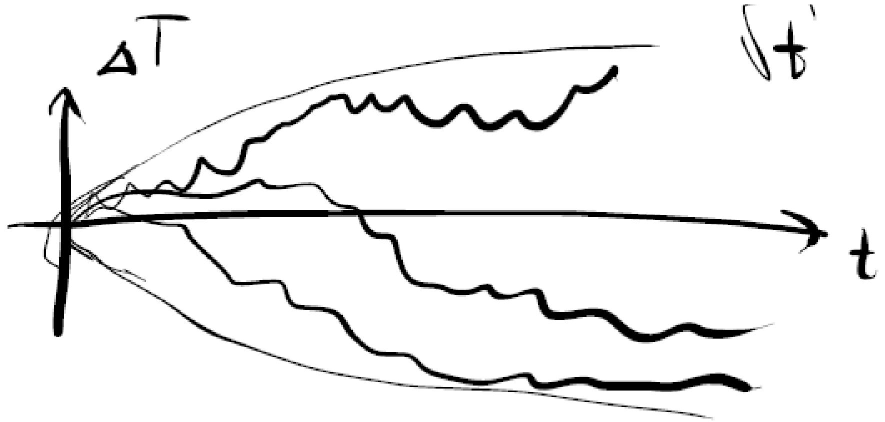
$\sigma_y^2 \sim \tau^{\mu}$



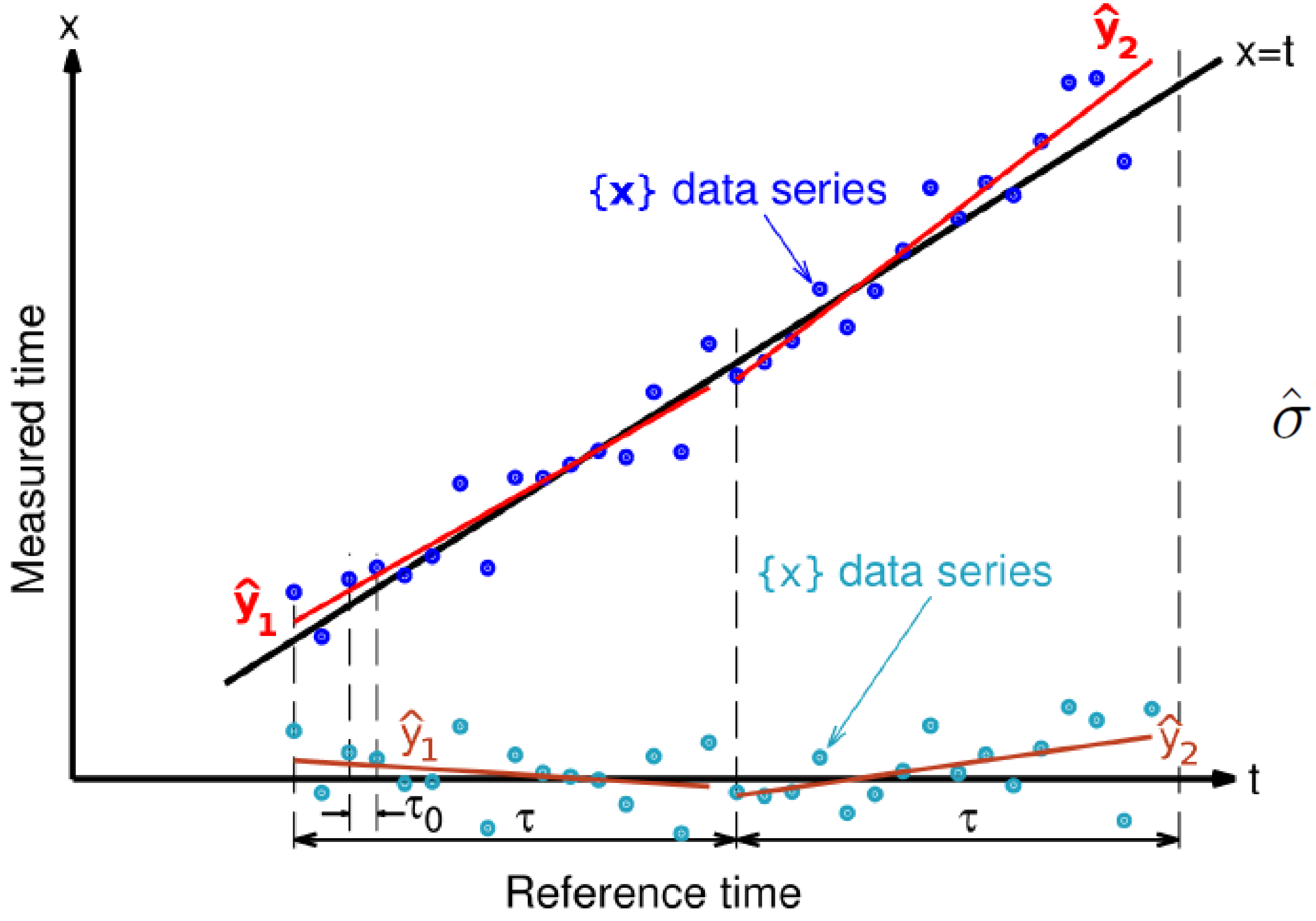
$\tau = m \cdot \Delta t$

$$\Delta T = T_o + (\Delta f/f)\cdot t + ½\, D\cdot t^2 + \sigma_x(t)$$

↖ модель ошибки атомных часов

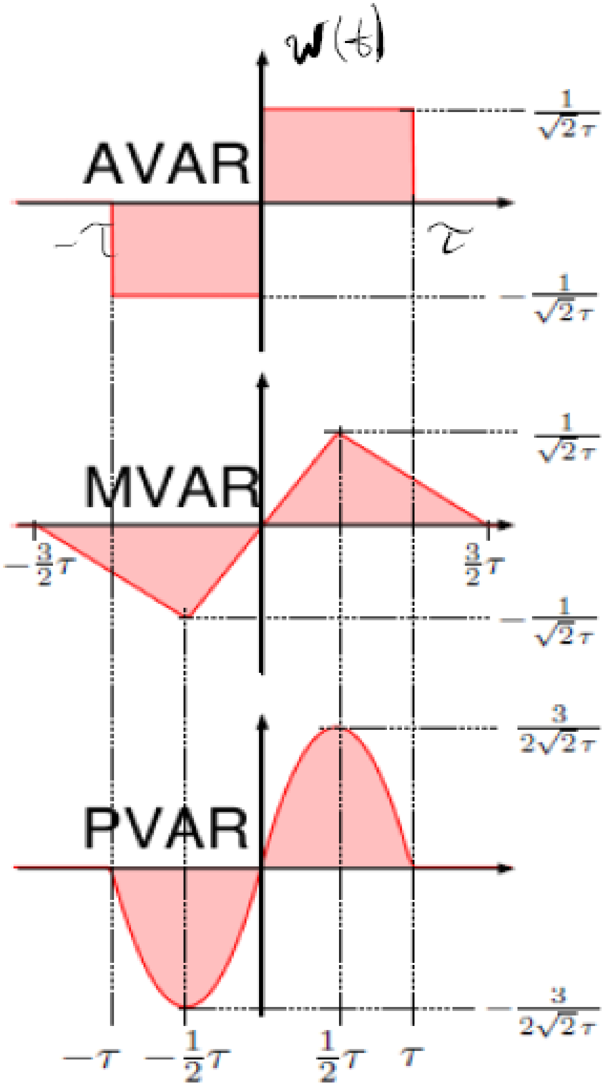


Параболическая вариация



$$\hat{\sigma}^2(\tau) = \frac{1}{2}\left\langle (\hat{y}_i - \hat{y}_{i+1})^2 \right\rangle$$

Вариации как результат вейвлет-преобразования



$$\sigma_y^2(\tau) = \left\langle \left(\int_{-\infty}^{\infty} y(t') h_\tau(t - t')\, dt' \right)^2 \right\rangle = \left\langle \left(\int_{-\infty}^{\infty} y(t) w(t' - t)\, dt' \right)^2 \right\rangle$$

$$T(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) w^*\left(\frac{t' - b}{a}\right) dt'$$

$b = t$, $a = \tau$ $\quad w(t) = h_\tau(-t)$

Francois Vernotte, Michel Lenczner, Pierre-Yves Bourgeois, and Enrico Rubiola
The Parabolic Variance (PVAR), a Wavelet Variance Based on the Least-Square Fit

Спасибо за внимание, продолжение следует….